Conformal Mapping P.G-sem-2nd (4)

paper-VI, unit-IV

Conformal Mapping

Introduction: We know that a bilinear transformation maps the points in the z-plane (i.e. xy-plane) onto ω-plane (i.e. uv-plane). We now investigate in more general terms the character of this transformation when the function $w = u(x,y) + iv(x,y)$ is analytic,

i.e. $u+iv = u(x,y)+iv(x,y)$ is analytic.

the set of equations

$$\left.\begin{array}{l} u = u(x,y) \\ v = v(x,y) \end{array}\right\} \longrightarrow (1)$$

$w = \frac{az+b}{cz+d}$ (1) $i = \frac{a0+b}{c0+d}$ (2)

From (3) $\beta = a$, from (1) $d = b/i = \alpha$ and from (2) $\gamma = i\alpha$.

$\therefore w = \frac{az+\alpha}{iaz-ia} = -i\left(\frac{z+1}{z-1}\right)$

defines a transformation or a mapping which establishes a correspondence between the points in the uv and xy plane. The equations (1) are called transformation equations. If each point of xy-plane is associated with a unique point of uv-plane and conversely. Then we say that this transformation or mapping is ~~one-to-one~~ one-one onto and ~~hence~~ the inverse of this trans-formation $x = x(u,v),\ y = y(u,v)$ — (2) is often *** exist. The condition for this, as we know from ~~difference~~ differential ~~equation~~ calculus, is that Jacobian of the transformation

viz. $$J = \frac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix} \neq 0$$

$$= u_x v_y - u_y v_x \neq 0$$

$\cancel{= u_x u_x - (-v_x)(v_x)}$

*** called the inverse transformation of (1)

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	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30				

Since $w = f(z)$ is assumed to be analytic, therefore u and v must satisfy Cauchy-Riemann equations.
i.e, $u_x = v_y$ and $u_y = -v_x$

Hence substituting into the Jacobian we have

$$J = \frac{\partial(u,v)}{\partial(x,y)} \quad \begin{vmatrix} \frac{\partial u}{\partial x} & -\frac{\partial v}{\partial x} \\ \frac{\partial v}{\partial x} & \frac{\partial u}{\partial y} \end{vmatrix}$$

$$= u_x v_y - u_y v_x$$

$$= u_x u_x - (-v_x)(v_x)$$

$$= (u_x)^2 + (v_x)^2$$

$$= \left|\frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x}\right|^2$$

$$= |f'(z)|^2 \neq 0$$

Hence ~~the~~ if $f(z)$ is analytic, the transformation $w = f(z)$ will have a single valued inverse in the neighbourhood of z, ~~~~ if $f'(z) \neq 0$. Exceptional points where $f'(z) = 0$ are known as critical points.